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OPTIMUM PROCESSING IN RANGE TO IMPROVE ESTIMATES OF DOPPLER AND POLARIMETRIC VARIABLES ON WEATHER RADARS

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1. INTRODUCTION

Radar meteorology applications rely on pulsed weather radars that survey the atmosphere and estimate a set of meteorological variables for each resolution volume. The reliability of these applications depends on the quality of the supplied data. To reduce the statistical uncertainty of estimates of spectral moments and polarimetric variables for each resolution volume, weather radars often average signals from many pulses. Therefore, the variance reduction of averaged estimates is inversely proportional to the equivalent number of independent samples M_I. This number is given by

$$M_I = \left(\sum_{m=-M+1}^{M-1} \frac{M-|m|}{M^2} \rho^2(m) \right)^{-1}, \qquad (1)$$

where the correlation coefficient $\rho(m)$ refers to sample time (from pulse to pulse) or range time, and m is an integer indicating the lag (Doviak and Zrnic, 1993). For correlation in sample time, the lags are mT_s, where T_s is the pulse repetition time; for sampling in range the lags are $m(\tau/L)$, where τ is the pulse length and L a positive integer greater than one if the pulse is oversampled in range.

The number of samples available for averaging is determined by the pulse repetition time and the dwell time. In addition to averaging along sample time, some radars average a few samples along range time to further reduce the estimates' errors. However, this process of simple averaging does not yield the best performance when the observations are correlated, and degrades the range resolution of the system, diminishing its effectiveness for sampling small-scale phenomena.

A well-known method to reduce the acquisition time without sacrificing range resolution is the pulse compression technique (Nathanson, 1969). Pulse compression can be applied to increase the number of independent samples by averaging high-resolution estimates in range. However, most ground-based weather radars do not use pulse compression due to the so-called range sidelobes and, more importantly, due to the need for increasing the transmission bandwidth.

This work presents a technique that increases the number of independent samples by keeping the dwell time constant without degrading the range resolution. More independent samples could reduce the estimates' errors at the same antenna rotation rate, or could speed up volume scans while keeping the errors at previous levels; in both cases there is no degradation in the range resolution of estimates.

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2. THE WHITENING TRANSFORMATION

It is know from estimation theory that classical estimators of the mean and variance of white (i.e. uncorrelated) Gaussian observations attain the Cramer-Rao lower bound, which is an indication of the theoretical best possible performance of an estimator. Therefore, if averaging were used to reduce the statistical uncertainty of estimators, one would like to derive a transformation on the original weather data based on its correlation coefficient such that the resulting samples are uncorrelated. Still, this transformation has to preserve the same properties that are of interest in the original sequence. Such transformation usually termed as "whitening" (Van Trees, 1968) exists if the underlying samples have zero mean.

It is somewhat surprising that previous works overlooked the fact that while the correlation of samples separated by T_s needs to be estimated for each particular case (it depends on the meteorological conditions being observed), samples spaced in range exhibit a correlation coefficient that allows its exact computation a priori; the underlying assumption here is that the mean echo power changes very little over the average interval in range. By exactly knowing the correlation coefficient, it is possible to apply the whitening transformation avoiding the pitfalls encountered if this correlation has to be estimated. As a result, we obtain $M_I = M$ [see (1)], and the variance reduction through averaging is maximized.

Maximization of the equivalent number of independent samples brings up the following implications:

- *For the same uncertainty as the one obtained with correlated samples, faster scan rates are possible, as the total number M of samples for a resolution volume is determined by the pulse repetition time and the dwell time.*
- *For the same scanning rates, lower uncertainties can be obtained, making the use of polarimetric variables feasible for accurate rainfall estimation and hydrometeor identification.*

With the advent of digital receivers (Brunkow, 1999), oversampling is indeed feasible. Therefore, it is possible to maintain the same range resolution and current radar capabilities, while adding, in parallel, a set of more reliable estimates.

The procedure starts with oversampling in range so that there are L samples during the pulse duration τ, (that is oversampling by a factor of L). Assume that the range of depth $c\tau/2$ (where c is the velocity of light) is uniformly filled with scatterers. For relatively short pulses this is a common occurrence, and the correlation coefficient is determined by the pulse shape and the receiver filter impulse response as (Torres, 2001)

$$\rho(m) = \left[\sum_{j=0}^{L-1} p^2(j)\right]^{-1} \left[\sum_{j=0}^{L-1} p(j)p(j-m)\right] * h(m) * h^*(-m)\,, \quad (2)$$

where the superscript * denotes complex conjugation, $p(m)$ is the transmitted pulse shape, and $h(m)$ is the impulse response of the receiver filter. Note that the expression for the correlation coefficient along range time depends solely on parameters that are known (or can be measured) and therefore allows for its exact determination.

The procedure for implementing the whitening transformation is as follows. Define the Toeplitz symmetric correlation coefficient matrix **C** (1 on the main diagonal, $\rho(1)$ on the first off diagonal, $\rho(2)$ on the second, etc.). Because this matrix is positive semi-definite, it can be decomposed into a product of a matrix **H** and its transpose as

$$\mathbf{C} = \mathbf{H}\,\mathbf{H}^{\mathbf{t}}. \quad (3)$$

Any **H** that satisfies (3) is called a square root of **C**, and is the inverse of a whitening transformation matrix $\mathbf{W} = \mathbf{H}^{-1}$, which if applied to the range samples $V(1,n)$, $V(2,n)$, ..., $V(L,n)$ (sample-time n is fixed) produces L uncorrelated random variables. Denote with $X(l,n)$, the sequence of range samples spaced T_s seconds apart each of which is obtained as

$$X(l,n) = \sum_{j=0}^{L-1} w_{l,j} V(j,n)\,, \quad (4)$$

where $w_{l,j}$ are the entries of **W**.

3. ESTIMATION OF SPECTRAL MOMENTS AND POLARIMETRIC VARIABLES

3.1. *Signal Power*

Power estimates for the horizontal (or vertical) polarization are computed from whitened oversampled data (from the corresponding polarization) as follows

$$\hat{S} = \frac{1}{ML} \sum_{l=0}^{L-1}\sum_{n=0}^{M-1} \left|X(l,n)\right|^2 - N\,, \quad (5)$$

where N is the noise power for the horizontal (or vertical) polarization, M is the number of samples obtained at a fixed range location, and L is the oversampling factor. It can be proved that the improvement in variance reduction ratio when compared with the regular processing where the whitening transformation is not applied is

$$\frac{Var[\hat{S}_{colored}]}{Var[\hat{S}_{whitened}]} = \frac{L^2+1}{2L}\,. \quad (6)$$

3.2. *Doppler Spectral Moments*

Autocovariance processing produces correlation estimates in sample time

$$\hat{R}_l(mT_s) = \frac{1}{M} \sum_{n=0}^{M-|m|-1} X^*(l,n) X(l,n+m)\,, \quad (7)$$

where $|m|$ is the lag index between 0 and M-1, and l the range index between 0 and L-1. $\hat{R}_l(mT_s)$ estimates are averaged in range (over the index l) so that the variance of the estimate

$$\hat{R}(mT_s) = \frac{1}{L} \sum_{l=0}^{L-1} \hat{R}_l(mT_s) \quad (8)$$

decreases as L increases as in (6).

From (8) Doppler velocity and spectrum width estimates can be obtained as

$$\hat{v} = -\frac{\lambda}{4\pi T_s} \arg\left\{\hat{R}(T_s)\right\}, \text{ and} \quad (9)$$

$$\hat{\sigma}_v = \frac{\lambda}{2\pi T_s \sqrt{2}} \left| \ln\left(\frac{\hat{S}}{\left|\hat{R}(T_s)\right|}\right) \right|^{1/2} \operatorname{sgn}\left[\ln\left(\frac{\hat{S}}{\left|\hat{R}(T_s)\right|}\right)\right], \quad (10)$$

respectively, where λ is the transmitter wavelength.

3.3. *Polarimetric Variables*

Denote with $V_V(l,n)$ the echo corresponding to the vertical polarization and with $V_H(l,n)$ the echo corresponding to the horizontal polarization and let the whitened signals (in range) be $X_V(l,n)$ for vertical and $X_H(l,n)$ for the horizontal polarization. If simultaneous transmission of H and V is employed (Zrnic, 1996) the powers S_H and S_V at the orthogonal polarizations are obtained as in (5) and their ratio yields the differential reflectivity.

The correlation is obtained as

$$\rho_{HV}(0) = \frac{\sum_{l=0}^{L-1}\sum_{n=0}^{M-1} X_H^*(l,n) X_V(l,n)}{\sqrt{\sum_{l=0}^{L-1}\sum_{n=0}^{M-1} \left|X_H(l,n)\right|^2} \sqrt{\sum_{l=0}^{L-1}\sum_{n=0}^{M-1} \left|X_V(l,n)\right|^2}}\,, \quad (11)$$

and the argument of (11) gives the total differential phase.

3.4. ***Performance with Additive Noise***

When applying the whitening transformation in the presence of additive noise, both signal and noise are evenly affected. The noise, which was white prior to the whitening transformation, becomes colored. It can be shown for a correlation matrix corresponding to an ideal transmitter/receiver system that the noise enhancement factor is

$$\frac{N_{whitened}}{N_{colored}} = \frac{L^2}{L+1} . \tag{12}$$

The previous equation shows that the noise is enhanced for L>1 (which is always the case when oversampling). Therefore, for weak SNR, the variance reduction achieved by increasing L will be masked by a corresponding noise power boost. This trade-off in the presence of additive noise makes the whitening transformation useful in cases of relatively large SNR (>12 dB, see Fig. 1). In fact, for weather radars, the SNR of signals from storms is indeed large.

4. CONCLUSIONS

A method for estimation of Doppler spectral moments and polarimetric variables on pulsed weather radars was presented. This scheme operates on oversampled echoes in range; that is samples of in-phase and quadrature phase components are taken at a rate several times larger than the reciprocal of the transmitted pulse length. The aforementioned radar variables are estimated by suitably combining weighted averages of these oversampled signals in range with usual processing of samples (spaced at pulse repetition time) at a fixed range location. The weights in range are chosen such that the oversampled signals become uncorrelated and consequently the variance of estimates decreases significantly (see Fig. 1). This variance reduction occurs only if the signal-to-noise ratios are relatively high as is usually the case for most signals in weather surveillance radars. At low signal-to-noise ratios the variances increase so that there are crossover points of the variances. Below the cross over SNR, the classical processing produces lower variances. In general the cross over SNR depends on the variable that is to be estimated and on some other parameters (spectrum width, number of samples, etc.) An objective decision on which estimates to use, classical or the ones obtained from whitened samples in range, should be based on the SNR and possibly on estimates of other parameters that affect the variance.

Because estimates' errors are inversely proportional to the volume scanning times, it follows that storms can be surveyed much faster than it is possible with current processing methods, or equivalently, for the current volume scanning time, accuracy of estimates can be greatly improved.

5. REFERENCES

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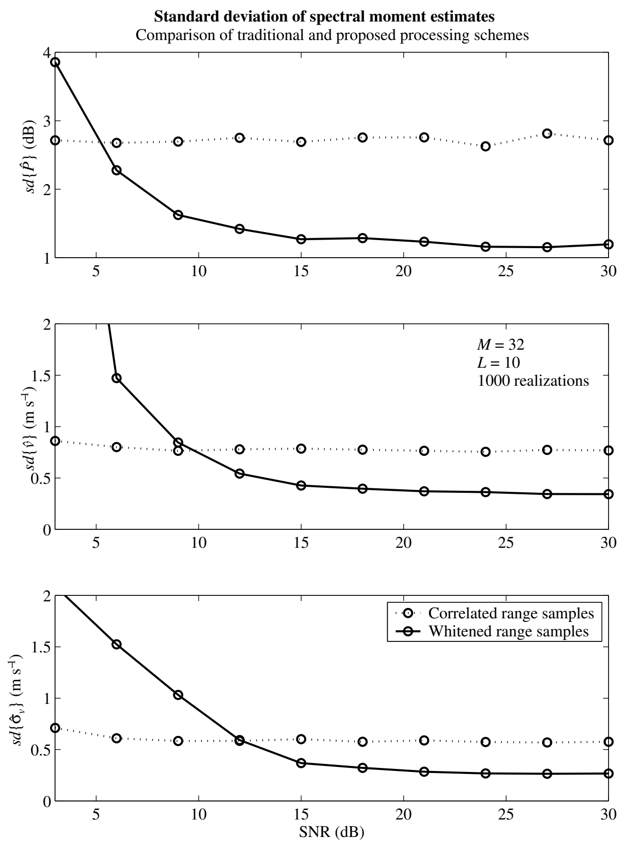

Figure 1. Standard deviation of power (top), mean Doppler velocity (middle), and Doppler spectrum width (bottom) obtained by simulating correlated range samples and applying both traditional and proposed processing. M is the number of time samples (separated by T_s) that are used to compute the Doppler spectrum and its moments. L is the oversampling factor, i.e., the number of range samples that are used to reduce the standard error of estimates. The simulation results were obtained from 1000 realizations.